LEBANESE AMERICAN UNIVERSITY
Division of Computer Science and Mathematics

Discrete Structures I

Exam I

Fall 2011 (October 28, 2011)

Name: KEY **ID:**

Circle the name of your instructor: Dr. S. Habre Dr. M. Hamdan

Question Number	Grade
1. 8%	
2. 10%	
3. 10%	
4. 16%	
5. 7%	
6. 10%	
7. 12%	
8. 15%	
9. 12%	
Total	

1. (8%) Show that the sum of two odd numbers is even. (Mention the proof type you are using.)

m odd $\Rightarrow m = 2k_1 + 1$

n odd $\Rightarrow n = 2k_2 + 1$

then $m+n = 2k_1 + 2k_2 + 2$

$= 2(k_1 + k_2 + 1)$

$= 2k_3 \Rightarrow m+n$ is even

this is a direct proof

2. (10%) Prove that if x is a **Rational number** and y is an **Irrational** number, then $x+y$ is **irrational**. (Mention the proof type you are using.)

Suppose $x+y$ is rational, then $x+y = \frac{m}{n}$.

But x is also rational $\Rightarrow$ $x = \frac{m'}{n'}$, then

$y = \frac{m}{n} - \frac{m'}{n'} = \frac{mn' - m'n}{nn'} = \frac{m''}{n''} \Rightarrow y$ is rational.

We have proven the contrapositive: $-q \rightarrow -p$.

3. (10%) Construct the truth table for the statement: $(q \wedge p) \vee (q \wedge \sim p)$

p	q	$q \wedge p$	$\sim p$	$q \wedge -p$	$(q \wedge p) \vee (q \wedge \sim p)$
T	T	T	F	F	T
T	F	F	F	F	F
F	T	F	T	T	T
F	F	F	T	F	F

4. (16%) Write the following statements in terms of p, q, and r; where

p : "I will study Discrete Structures"; q : "I will go to the movies"; r : "I am in a good mood."

(a) If I am in a good mood, then I will go to a movie

$r \to q$

(b) I will not go to a movie and I will not study Discrete structures

$-q \wedge -p$

(c) I will go to a movie **only if** I will not study Discrete structures

$q \to -p$

(d) If I will not study Discrete Structures then I am in a good mood

$-p \to +r$

5. (7%) Write the negation of the statement: $\forall x : P(x) \to Q(x)$. $(\forall x, -P(x) \vee Q(x))$

$\exists x, - (P(x) \vee Q(x))$

$\Rightarrow \exists x, P(x) \wedge \bar{-} Q(x)$.

6. (10%)Show **in two different ways** the equivalence of the following statements:

$$p \to (q \to r) \text{ and } (p \wedge q) \to r$$

$p \to (q \to r) \equiv -p \vee (q \to r) \equiv -p \vee [-q \vee r]$

$\equiv \cancel{p \vee (-q \wedge r)} \cancel{\equiv (p \vee q) \wedge (p \vee r)} \equiv -p \vee -q \vee r$

$\equiv -(p \wedge q) \vee r$

$(p \wedge q) \to r \equiv (p \wedge q) \vee (-r) \equiv (p \vee -r) \wedge (q \vee -r).$ $\equiv (p \wedge q) \to r$

p	q	r	~~$p \to$~~ $q \to r$	$p \to (q \to r)$
T	T	T	T	T
T	T	F	F	F
T	F	T	T	T
T	F	F	T	T
F	T	T	T	T
F	T	F	F	T
F	F	T	T	T
F	F	F	T	T

$p \wedge q$	$p \wedge q \to r$
T	T
T	F
F	T
F	T
F	T
F	T
F	T
F	F

7. (12%) Let $P(x, y) : y$ is a multiple of x,(that is, $x \mid y$) where $x, y \in \mathbb{Z}$.

(a) Translate into English the statement: $\forall x, \exists y : P(x, y)$

Every integer has a multiple.

(b) Write down the negation of the statement above in English

There is one integer that has no multiples.

(c) Write down the negation of the statement above in symbols

$\exists x, \forall y \; \neg P(x, y)$

8. (15%) Consider the statement: $P(n) : 1 + 3 + 5 + ... + (2n - 1)$

(a) Calculate $P(1), P(2), P(3), P(4)$.

$P(1) = 1$

$P(2) = 1 + 3 = 4$

$P(3) = 1 + 3 + 5 = 9$

$P(4) = 1 + 3 + 5 + 7 = 16$

(b) Deduce $P(n)$ in general in terms of n.

$P(n): 1 + 3 + 5 + \dots + (2n-1) = n^2$.

<u>Step 1</u>: ✓

<u>Step 2</u>: Suppose $P(k)$ is true

<u>Step 3</u>: Show $P(k+1)$ is true:

$1 + 3 + 5 + \dots + [2(k+1)-1] = 1 + 3 + 5 + \dots + (2k+1)$

$= 1 + 3 + 5 + \dots + (2k-1) + (2k+1)$

$= k^2 + (2k+1) = (k+1)^2$.

(c) Prove by induction your answer in part b

See previous page.

9. (12%) Show using induction that $4^n - 1$ is a multiple of 3 (that is, $3 | 4^n - 1$) for all $n = 1, 2, 3, ..$

for $n = 1$, $4 - 1 = 3$ ✓

Suppose $4^k - 1 = 3r$.

Show: $4^{k+1} - 1 = 3r'$.

$$4^{k+1} - 1 = 4^k \cdot 4 - 1$$
$$= 4^k(3+1) - 1$$
$$= 3 \cdot 4^k + \underbrace{4^k - 1}_{3r}$$
$$= 3(4^k + r) = 3r'$$